# Control System Design

Lecture 2

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Elective Course in Mechatronics Engineering
Credits (2/2/3)

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## Block Diagrams: Description

### Characteristics of the Block Diagram

*A block diagram is a graphical representation of the cause-effect relationship between signals by blocks and directed lines*

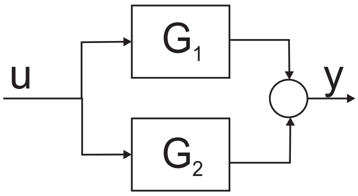
$\Rightarrow$ Visualization of direction of action and interdependencies

### Block Diagram Components

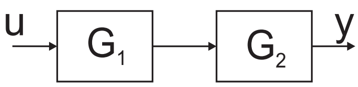
- Directed Lines: System signals and their direction of action
- Circles: Summation of signals
- Rectangles: Dynamic relationship between signals; in our case by transfer functions
  $\Rightarrow$ Rectangles and circles are transfer blocks

# Block Diagrams: Connection Rules

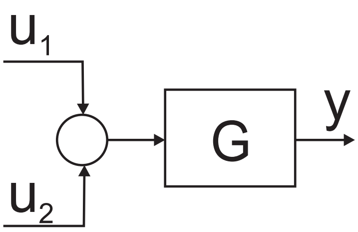
**Parallel**




**Series**




**Summation 1**




**Equivalent Representation**

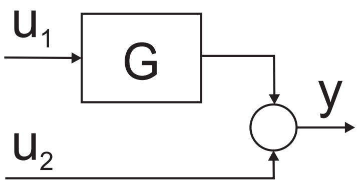
Gap 1



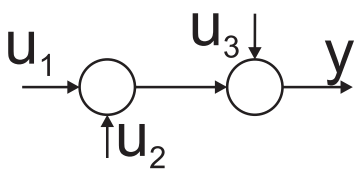
Block Diagrams Feedback Loop Linear State Space Models

# Block Diagrams: Connection Rules

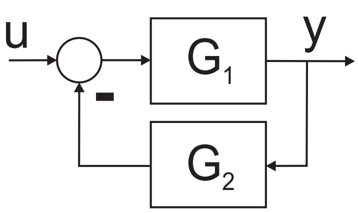
**Summation 2**




**Summation 3**




**Feedback**




**Equivalent Representation**

Gap 2

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# Block Diagrams: Simplification Example

## Simplification

Gap 3





# Feedback Loop: Basics

## Block Diagram

Gap 4

## Explanation

- Input signal: $u$; output signal: $y$
- Reference signal: $r$; error signal $e = r - y$
- Output disturbance: $d$; input disturbance: $d_{\mathrm{i}}$
- Plant transfer function $G$
- Controller transfer function $C$

# Feedback Loop: Sensitivities

## Example

Gap 5





# Feedback Loop: Stability

## Sensitivities

- Complementary Sensitivity: $T(s) := \dfrac{Y(s)}{R(s)} = \dfrac{C(s)G(s)}{1 + C(s)G(s)}$
- Sensitivity: $S(s) := \dfrac{Y(s)}{D(s)} = \dfrac{1}{1 + C(s)G(s)}$
- Input Disturbance Sensitivity: $S_{\mathrm{i}}(s) = \dfrac{Y(s)}{D_{\mathrm{i}}(s)} = \dfrac{G(s)}{1 + C(s)G(s)}$
- Control Sensitivity: $S_u(s) = \dfrac{U(s)}{R(s)} = \dfrac{C(s)}{1 + C(s)G(s)}$

## Stability Test

*All signals in the feedback loop should remain bounded*

$\Rightarrow$ Internal stability: all sensitivities must be stable

## Equivalent Stability Test

- $1 + G(s)\,C(s)$ must only have zeros in the OLHP

# Feedback Loop: Stability

## Example

Gap 6





# Feedback Loop: Reference Tracking

## Desired Behavior

- Consider $\dfrac{Y(s)}{R(s)} = T(s) = \dfrac{C(s)\,G(s)}{1 + C(s)\,G(s)}$
  $\rightarrow$ Ideally, the output signal should exactly follow (track) the reference signal: $y(t) = r(t)$ and $T(s) = 1$
  $\rightarrow$ In practice, we try to design $C(s)$ such that $T(s) \approx 1$

Gap 7

# Feedback Loop: Reference Tracking

**Steady-state Error** $\lim_{t\to\infty} e(t)$ **after Reference Steps**

*The steady state error should converge to zero or at least be small*

**Final Value Theorem of the Laplace Transform**

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$$

**Final Value of the Step Response** (stable $G$)

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0} s\,G(s)U(s) = \lim_{s\to 0} s\,G(s)\frac{1}{s} = G(0)$$

**Requirement for the Feedback Loop**

$$T(0) = 1 \text{ or at least } T(0) \approx 1$$





# Feedback Loop: Disturbance Rejection

**Output Disturbance**

- Consider $\dfrac{Y(s)}{D(s)} = S(s) = \dfrac{1}{1 + C(s)\,G(s)}$
  $\rightarrow$ Ideally, the output signal should not change in case of a disturbance: $y(t) = 0$ and $S(s) = 0$
  $\rightarrow$ In practice, we try to design $C(s)$ such that $S(s) \approx 0$

Gap 8

# Feedback Loop: Disturbance Rejection

## Input Disturbance

- Consider $\frac{Y(s)}{D_i(s)} = S_i(s) = \frac{G(s)}{1 + C(s)\,G(s)}$

Gap 9

## Output Disturbance Step $d(t) = \sigma(t)$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} S(s) \approx 0$$

## Input Disturbance Step $d_{\mathrm{i}}(t) = \sigma(t)$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} S_{\mathrm{i}}(s) \approx 0$$



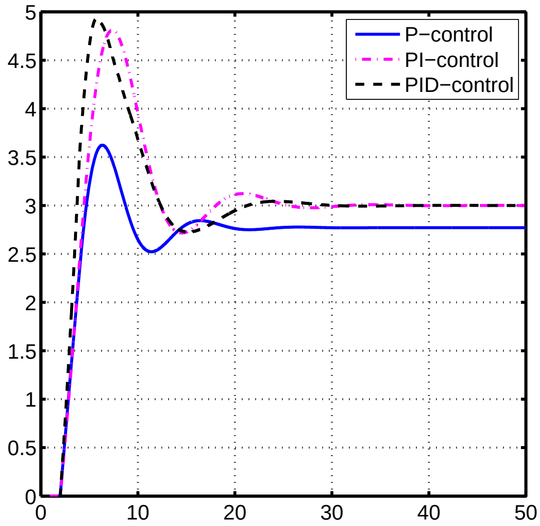


# Feedback Loop: Desired Behavior

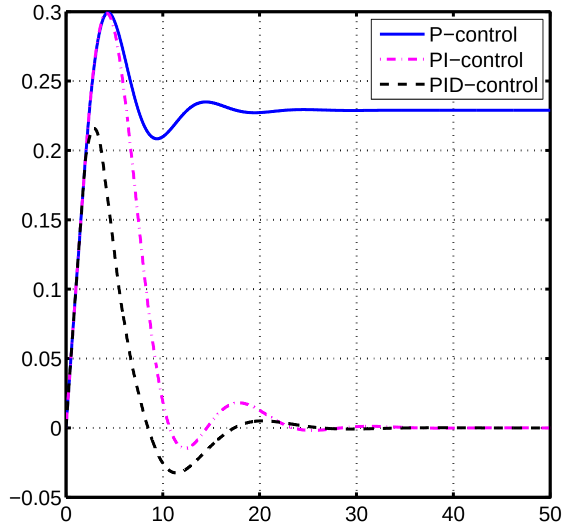
## Example

- Temperature control system from ECE388

## Reference Step Response




## Disturbance Step Response



# Linear State Space Models: Definitions

## State Space Equations

$$\dot{x}(t) = A\,x(t) + b\,u(t) + o\,w(t)$$
$$y(t) = c^T\,x(t) + d\,u(t)$$

## Signals

- State vector: $x(t) \in \mathbb{R}^n$
- State derivative: $\dot{x}(t) \in \mathbb{R}^n$
- Input: $u(t) \in \mathbb{R}$
- Output: $y(t) \in \mathbb{R}$
- Disturbance: $w(t) \in \mathbb{R}$

## Constant Matrices and Vectors

- Dynamics matrix: $A \in \mathbb{R}^{n\times n}$
- Input vector: $b \in \mathbb{R}^{n\times 1}$
- Disturbance vector: $o \in \mathbb{R}^{n\times 1}$
- Output vector: $c^T \in \mathbb{R}^{1\times n}$
- Feed-through: $d \in \mathbb{R}$

## Transfer Function

$$G(s) = c^T(sI - A)^{-1}b + d$$





# Linear State Space Models: Example

## Computation

Gap 10