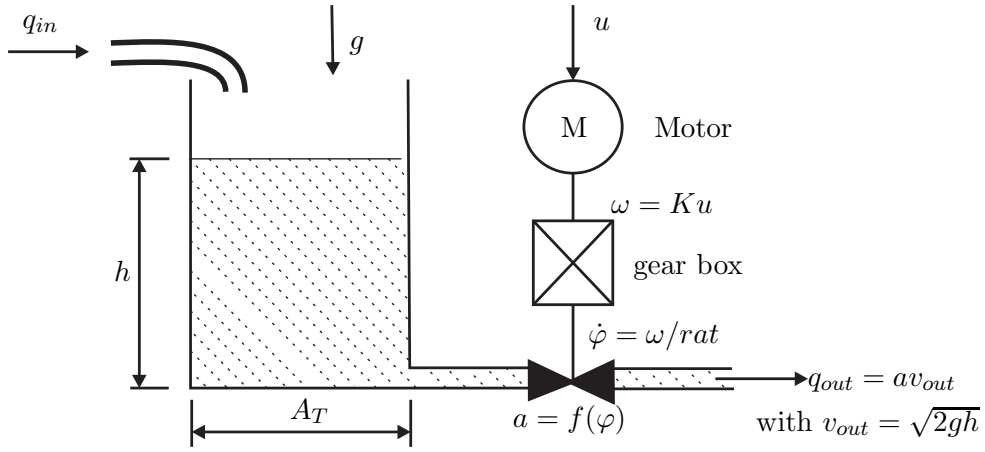
Exercise Sheet 3:

Problem 7:

We are given the cylindric tank system with the surface area A_T that is shown in the following figure. It is filled with water up to a height h. For control purposes, it is possible to influence h by changing the position of a valve that regulates the outflow q_{out} of the tank. There is no possibility to influence the inflow q_{in}.



The actuation of the valve is performed by a motor that turns with an angular velocity ω depending on the voltage u. The relation between ω and u is described as $\omega = Ku$, whereby K is a constant. In addition, a gear box translates ω into a smaller angular velocity $\dot{\varphi}$ depending on the gear box ratio rat. That is, $\dot{\varphi} = \omega/rat$. The outflow q_{out} can be written as the product of the valve area a and the outflow velocity v_{out}, that is $q_{out} = a \cdot v_{out}$. There is a nonlinear dependency between a and the angle φ that is given by the characteristic curve on the second page. In addition, there is a nonlinear dependency between the outflow velocity v_{out} and the water height h that is given by $v_{out} = \sqrt{2\,g\,h}$ (g is the gravitational acceleration).

a. Develop a nonlinear state space model that describes the dynamic relationship between the input voltage u, the inflow q_{in}, the outflow q_{out} and the water level height h.

Hint: Use the following information:

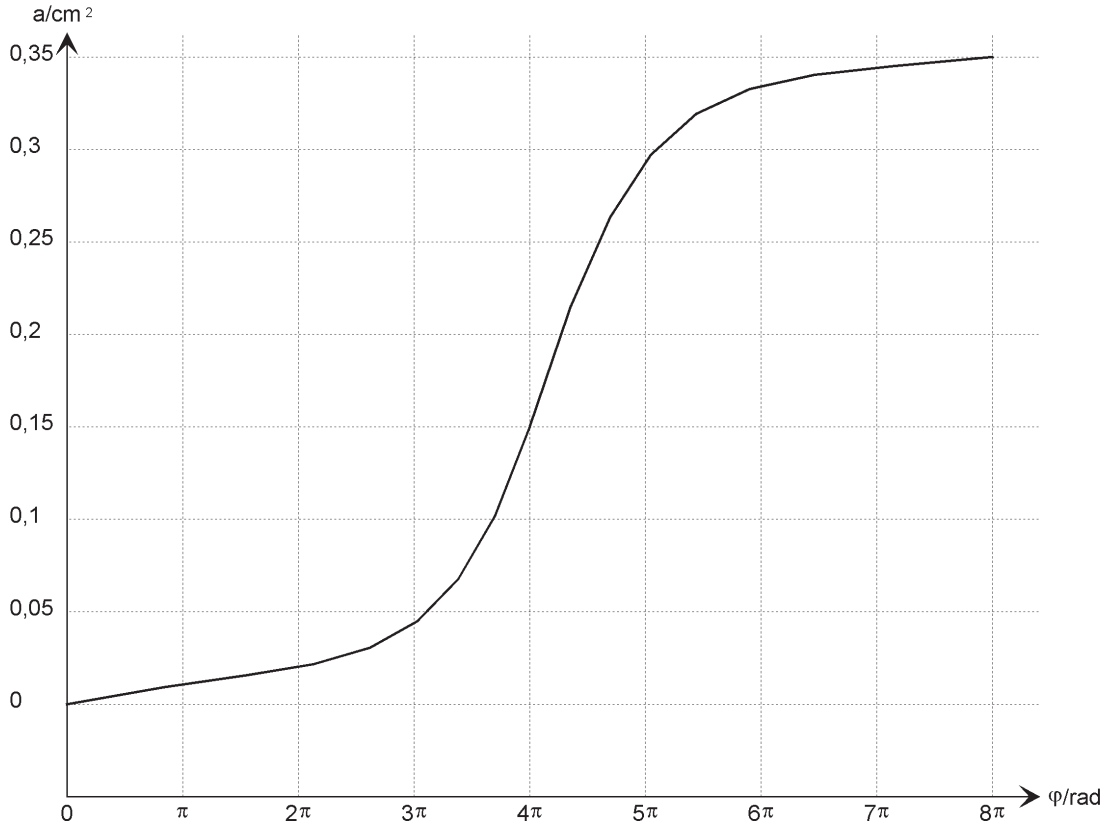
- h and φ are the state variables
- Volume flow: $A_T \cdot \dot{h} = q_{in} - q_{out}$ and $q_{out} = a \cdot \sqrt{2\,g\,h}$
- $\dot{\varphi} = \omega/rat$ and $\omega = K \cdot u$

From now on, we will use the parameter values $K = 7.5\,\mathrm{rad/(sec{\cdot}V)}$, $g = 10\,\mathrm{m/sec^2}$, $A_T = 150\,\mathrm{cm^2}$, $rat = 15$.

b. Determine the set-point of the tank system for the case that $q_{in,SP} = 50\,\mathrm{cm^3/sec}$ and $\varphi_{SP} = 4\pi$.

Hint: You should use the characteristic curve in the figure below to find the set-point value of the valve surface a_{SP}.

c. Linearize the system around this set-point and write down the linear state space model



Problem 8:

a. Draw a block diagram of the tank system in Problem 7 with two integrators.

b. Simulate the tank system in Simulink with $h(0) = h_{SP}$, $q_{in}(t) = q_{in,SP}$, $\varphi(0) = \varphi_{SP}$ and $u(t) = u_{SP}$. What do you observe?

Hint: You will find the nonlinear function $f(\varphi)$ in the file `ECE441_Ex3_nonlinear_function.mdl` on the course webpage.

c. Use the Simulink model in **b.** Apply an input step $u(t) = u_{SP} + \sigma(t)$ and measure the difference variable $h(t) - h_{SP}$.

d. Simulate an input step $u(t) = \sigma(t)$ for the linearized model in Problem 7 **c.** Compare the output signal to the simulation in **c.**