Control System Design

Lecture 10

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Mechatronics Engineering
Credits (2/2/3)

Webpage: http://mece441.cankaya.edu.tr

Laplace Transform: General Formula

Fact from Laplace Transform

- Assume that $h(t) \circ\!\!-\!\!\bullet H(s)$ and $H(s)$ has positive relative degree and its region of convergence is $s > -\alpha$. Then for any $z_0 > -\alpha$

$$\int_0^\infty h(t)e^{-z_0 t}dt = \lim_{s \rightarrow z_0} H(s)$$

Evaluation

Gap 1

General Zeros and Poles: Notation

Control Loop

Gap 2

Assumptions

- All closed loop poles at the left of $s = -\alpha < 0$ (stable closed loop)
- $G(s) = \frac{B(s)}{A(s)} = \frac{(s - z_0)B'(s)}{(s - p_0)A'(s)}$, $\mathrm{Re}(z_0) > -\alpha$ and $\mathrm{Re}(p_0) > -\alpha$
  $\Rightarrow$ plant zero at z_0 and plant pole at p_0
- $C(s) = \frac{P(s)}{L(s)} = \frac{P(s)}{sL'(s)}$
  $\Rightarrow$ Integral control

General Zeros and Poles: Properties

Plant Zero at z_0 and Reference or Output Disturbance Step

$$\int_0^{\infty} e(t)e^{-z_0 t}dt = \frac{1}{z_0}$$

Gap 3

Implication

- For $z_0 \in \mathbb{R}$ and $z_0 < 0$, $e(t)$ must change sign $\Rightarrow$ overshoot

General Zeros and Poles: Properties

Plant Pole at p_0 and Reference or Output Disturbance Step

$$\int_0^\infty e(t)e^{-p_0 t}dt = 0$$

Computation

Gap 4

Implication

- For any p_0, $e(t)$ must change sign $\Rightarrow$ overshoot

Laplace Transform General Zeros and Poles Motivation Sensors

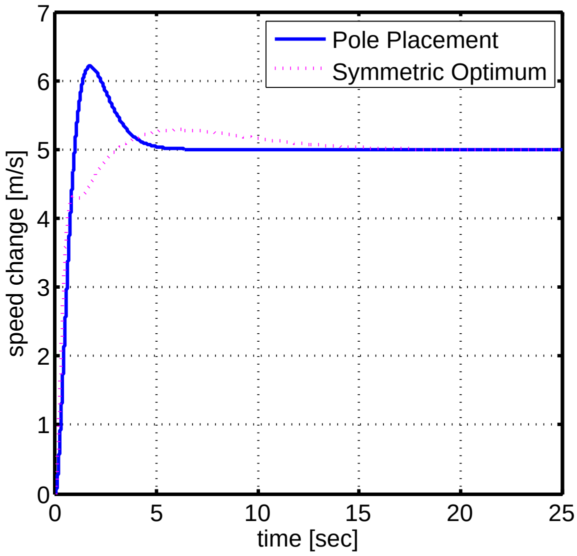
General Zeros and Poles: Properties

Vehicle Control Example

Gap 5

Simulation

- $-\alpha = -2$
- $p_0 = -1/19.5 = -0.05 > -\alpha$



Klaus Schmidt Department
Department of Mechatronics Engineering – Çankaya University

General Zeros and Poles: Properties

Plant Zero z_0 and Reference Step

$$\int_0^\infty y(t)e^{-z_0 t}dt = 0$$

Implication

- If $\mathrm{Re}(z_0) > 0$, then $y(t)$ must first be negative since it is finally positive

$\Rightarrow$ Undershoot for reference step

Temperature Control Example

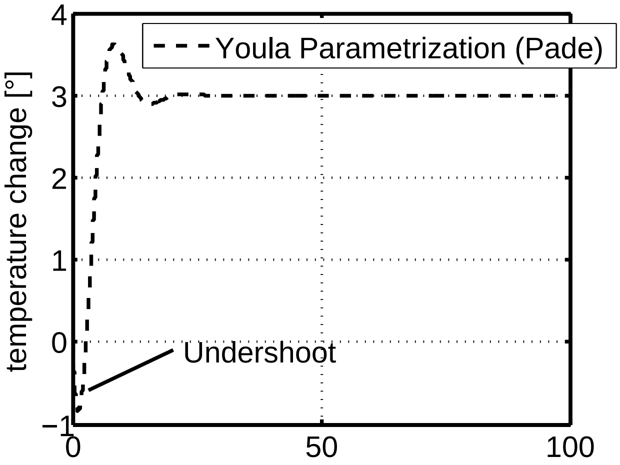
Gap 6

Temperature Control Example

- Padé-approximation:

$$G(s) = \frac{5(1-s)}{(1+s)(1+30s)}$$

$\Rightarrow z_0 = 1 > 0$



Summary: Performance Trade-offs

Main Observations

- If $|-\alpha|$ is larger than the smallest plant zero with positive real part, there will be large undershoot
- If $-\alpha$ is smaller than the smallest plant zero with negative real part, then significant overshoot will occur
- If $|-\alpha|$ is smaller than the real part of the largest instable open loop pole, then significant overshoot is expected
- If $-\alpha$ is smaller than the real part of the smallest stable open loop pole, then there will be overshoot

Summary: Illustration

Pole-Zero Diagrams

Gap 7

Laplace Transform General Zeros and Poles Motivation Sensors

Summary: Illustration

Pole-Zero Diagrams

Gap 8

Fundamental Limitations: Sources

Basic Feedback Loop

Gap 9

Sources for Limitations

- Sensors
- Actuators
- Disturbances/Noise

Sensors: Measurement Noise

Noise Transfer Function

$$Y(s) = -T(s)N(s)$$

Noise Properties

- Usually high frequency signal
  $\Rightarrow N(j\omega) = 0$ for $\omega < \omega_N$

Noise Attenuation

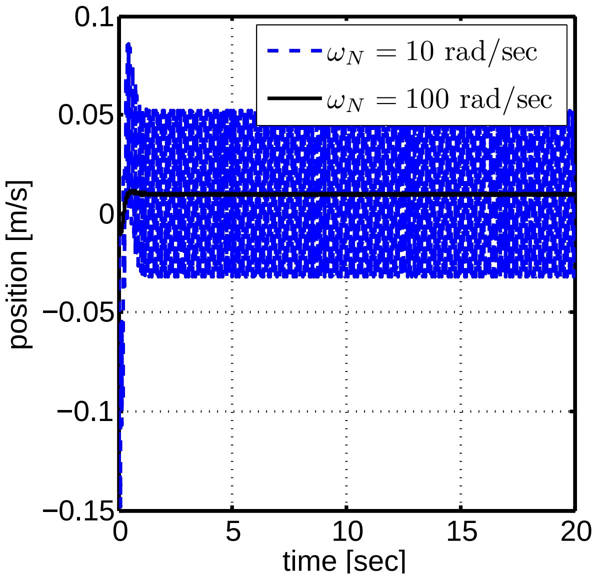
- $|T(j\omega)|$ should be small where $N(j\omega)$ is significant

  $\Rightarrow$ Poles of $T(j\omega)$ should be sufficiently smaller than ω_N

  $\Rightarrow$ Noise poses limit for closed-loop dynamics

Magnetic Suspension Example

- Closed-loop poles at $s = -10\,\text{rad/sec}$



Sensors: Sensor Response

Sensor Delay/Lag

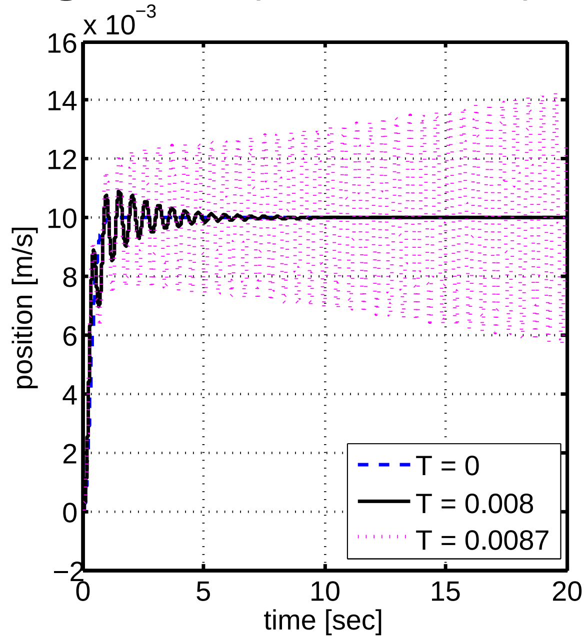
- Delayed measurement depending on sensor realization

$$Y(s) = \frac{1}{1 + s\,T} Y'(s)$$

Gap 10

$\Rightarrow$ Sensor dynamics can dominate plant dynamics if T is large

Magnetic Suspension Example



- Lag can lead to instability

Sensors: Sensor Response

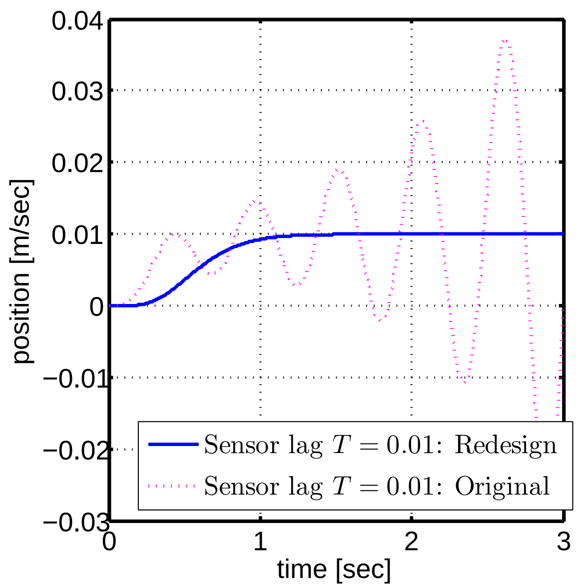
Possible Solutions

- Use higher-quality sensor
  $\Rightarrow$ Might be expensive
- High-pass filter for y

$$Y_m(s) = \frac{1 + sT}{1 + s\tau} Y(s), \quad \tau << T$$

  $\Rightarrow$ Might amplify high-frequency noise
- Consider sensor dynamics as part of plant
  $\Rightarrow$ Perform controller design including sensor dynamics

Magnetic Suspension Example



- Redesign achieves stable closed loop