Control System Design

Lecture 13

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Elective Course in Mechatronics Engineering
Credits (2/2/3)

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Feedforward Control: Basic Situation

Plant Transfer Function

$$\frac{Y(s)}{U(s)} = G(s) = \frac{B(s)}{A(s)}$$

Transfer Block

Gap 1

Design Goal

- Trajectory tracking: plant output $y(t)$ should follow desired output signal $y_d(t)$
  $\Rightarrow$ Compute required input signal $u(t)$ such that $y(t) = y_d(t)$

Illustration

Gap 2

Feedforward Control: Explanation

Simple Examples

Gap 3

Feedforward Control: Input Computation

Differential Equation From $G(s)$

$$(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0) Y(s) = (b_n s^n + b_{n-1} s^{n-1} + \cdots + b_0) U(s)$$

$$\Rightarrow a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = b_n u^{(n)} + b_{n-1} u^{(n-1)} + \cdots + b_0 u$$

Computation of $u(t)$

- Desired trajectory $y(t) = y_d(t)$ is given
- Solve the following differential equation to compute u

$$\Rightarrow b_n u^{(n)} + b_{n-1} u^{(n-1)} + \cdots + b_0 u = a_n y_d^{(n)} + a_{n-1} y_d^{(n-1)} + \cdots + a_0 y_d$$

Conditions

- Desired trajectory $y_d(t)$ has to be differentiable n times
- $y_d(t)$ should be bounded such that u stays bounded
- Assume that $B(s)$ does not have any instable zeros

Feedforward Control: Vehicle Control Example

Example

Gap 4

Feedforward Control: Vehicle Control Example

Example

Gap 5

Trajectory Planning: Trajectory Choice

Common Situation: Set-point Change

- $y_d(t) = y_0$ for $t \leq 0$
- $y_d(t) = f(t)$ for $0 \leq t \leq t_f$
- $y_d(t) = y_f$ for $t \geq t_f$

$\Rightarrow y_d(t)$ smoothly changes its value from y_0 to y_f between time 0 and a final time t_f

Illustration

Gap 6

Trajectory Planning: Example Trajectory

Polynomial

$$f(t) = v_0 + v_1\,t + v_2\,t^2 + \cdots + v_l\,t^l$$

Requirements

- $y_d(t)$ must be n times continuously differentiable
  $\Rightarrow y_d^{(i)}(0) = 0$ and $y_d^{(i)}(t_F) = 0$ for $i = 1, \ldots, n$
- $y_d(t) = y_0$ for $t \leq 0$
- $y_d(t) = y_f$ for $t \geq t_f$

$\Rightarrow$ We need $l = 2 \cdot (n+1) - 1 = 2\,n + 1$

Coefficients

- Write equations for above requirements
  $\Rightarrow l$ linear equations with l unknowns $v_0, \ldots, v_l$
- Obtain coefficients from solution of linear equations

Trajectory Planning: Verification of Conditions

Computation

Gap 7

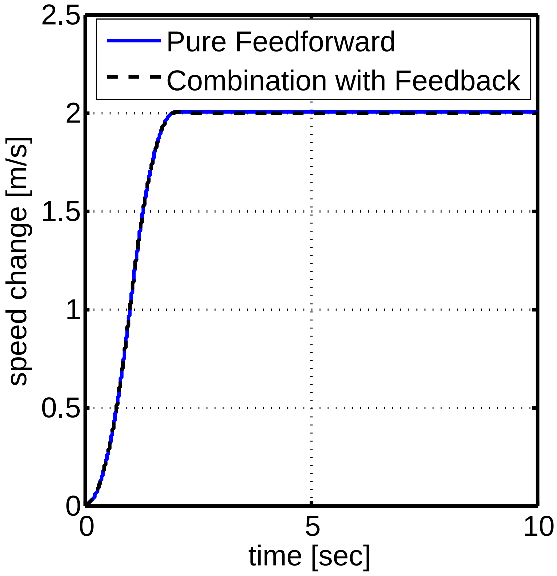
Trajectory Planning: Example

Example

Gap 8

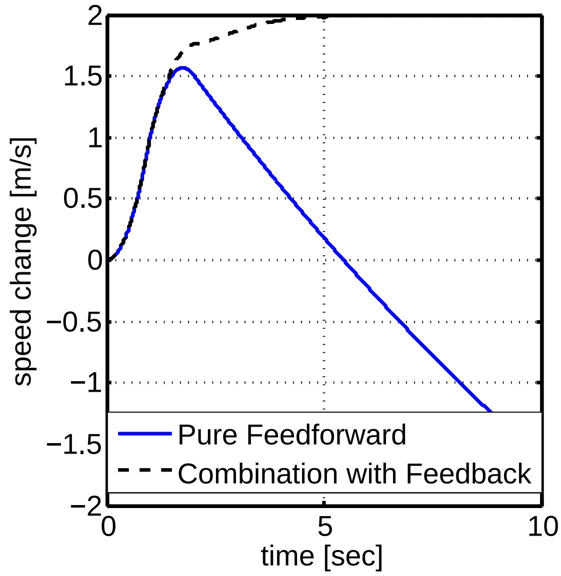
Feedforward Control: Vehicle Control Example

No Disturbance



⇒ Exact trajectory tracking due to feedforward

Disturbance Step



⇒ No disturbance compensation if feedforward control is used!

Feedforward Control: Combination with Feedback Control

Block Diagram

Gap 9

Explanation

- Generate input u_d to achieve desired output $y_d(t)$
- Reject disturbances by feedback controller C with output u_f
  ⇒ Feedback controller C only acts in case of deviations from $y_d(t)$!