Control System Design

Lecture 1

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Mechatronics Engineering
Credits (2/2/3)

Webpage: http://MECE441.cankaya.edu.tr

Content and Structure

Content

- Control Basics
- Classical PID design methods
- Algebraic controller design methods
- Fundamental limits
- Control architectures
- Matlab/Simulink experiments

Structure

- 2 lecture hours: Wednesday 9:20 - 11:10
- 2 hours for Matlab laboratory and exercises: Friday 9:20 – 11:10
- Office hours: Wednesday: 11:10 – 12:10

Grading and Literature

Grading

- 10 Quizzes (10%)
- Problem Solving and Laboratory work (25%)
- 1 Midterm Exam (25%)
- 1 Final Exam (40 %)

Literature

- Goodwin, Graham, Graebe Stefan, Salgado, Mario: "Control System Design", Prentice-Hall, Inc., 2001 (ISBN: 0-13-958653-9) (Main Textbook)
- Astrom, Karl and Murray, Richard: "Feedback Systems: An Introduction for Scientists and Engineers", Princeton University Press, 2008 (ISBN: 0-691-13576-2)
- Killian, Christopher: "Modern Control Technology", Thompson Delmar Learning, 2005 (ISBN: 1-4018-5806-6)

Linear Systems: Definition

System

- Input signal u
- Output signal y
- Operator H that maps u to y

$$y(t) = H\{u(t)\}$$

Illustration

Linear System

- Input signal as superposition of different input signals

$$u(t) = \alpha_1 u_1(t) + \alpha_2 u_2(t), \quad (\alpha_1, \alpha_2 \in \mathbb{R})$$

- Output signal as superposition of corresponding output signals

$$y(t) = H\{\alpha_1 u_1(t) + \alpha_2 u_2(t)\} = \alpha_1 H\{u_1(t)\} + \alpha_2 H\{u_2(t)\}$$

Linear Systems: Examples

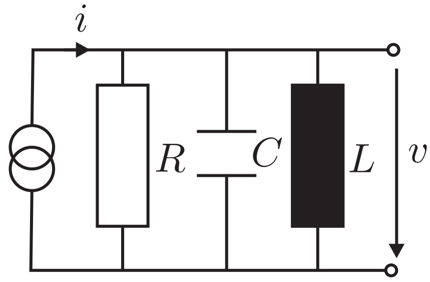
Illustration

Gap 1

Linear Systems: Modeling

RLC-Circuit Example

Gap 2



- Input: $u = i$
- Output: $y = v$

Differential Equation

$$C \cdot \frac{d^2 y}{dt^2} + \frac{1}{R} \cdot \frac{dy}{dt} + \frac{1}{L} \cdot y = \frac{du}{dt}$$

Linear Systems: General Model

Linear Time-invariant Ordinary Differential Equation

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_1 \dot{u} + b_0 u$$

Notation

- y: output signal (in the time domain)
- u: input signal (in the time domain)
- $y^{(i)}$ ($u^{(i)}$): i-th time derivative of y (u)
- a_i, b_i: coefficients in $\mathbb{R}$
- n: highest derivative of y
- m: highest derivative of u

Transfer Functions: Laplace Domain Representation

Laplace Transform

- Transformed input signal: $u \circ\!\!-\!\!\bullet\ \mathcal{L}(u) = U(s)$
- Transformed output signal: $y \circ\!\!-\!\!\bullet\ \mathcal{L}(y) = Y(s)$

Input/Output Relation

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} =: G(s) = \frac{B(s)}{A(s)}$$

Transfer Block Representation

- Input/output variables u, y
- Dynamic relation G

Transfer Functions: Examples

RLC-Circuit

Gap 3

Transfer Functions: Properties

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} =: G(s) = \frac{B(s)}{A(s)}$$

Poles

- Zeros of the denominator polynomial: s such that

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

Zeros

- Zeros of the numerator polynomial: s such that

$$B(s) = b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0 = 0$$

Relative Degree

- Difference between polynomial degrees of denominator and numerator

$$r = \deg A(s) - \deg B(s)$$

- $G(s)$ is called *proper* if $r \geq 0$

Transfer Functions: Properties

Example

Gap 4

Transfer Functions: Properties

Pole-Zero Diagram

Gap 5

Stability

A system is called (bounded input bounded output – BIBO) stable if any bounded input signal u leads to a bounded output signal y

Transfer Functions

- $G(s)$ is BIBO stable if all of its poles lie in the open left half plane of the complex plane

Transfer Functions: Properties

Example

Gap 6

Step Responses: Computation

Definition

The unit step response is the output response of a dynamic system to the Heaviside step function applied at its input

- Heaviside step function: $\sigma(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{otherwise} \end{cases}$

Gap 7

Step Response Computation

- Input: $u(t) = \sigma(t) \circ\!\!-\!\!\bullet\ U(s) = \frac{1}{s}$
- Output: $Y(s) = G(s)U(s) = G(s)\frac{1}{s}\ \bullet\!\!-\!\!\circ\ y(t) = \mathcal{L}^{-1}\big(G(s)\frac{1}{s}\big)$

Step Responses: Examples

Proportional Gain

$$G(s) = K$$

Integrator

$$G(s) = \frac{1}{s}$$

First-order Lag

$$G(s) = \frac{1}{1 + s\,T}$$

Second-order Lag

$$G(s) = \frac{\omega_n^2}{s^2 + 2\,D\,\omega_n\,s + \omega_n^2}$$

Gap 8

Transfer Functions: Summary and Remarks

Summary

- Mathematical plant model describes the dynamic input/output relation of technical processes
- This lecture mostly considers plants that can be modeled by linear time-invariant ordinary differential equations
- Laplace transform leads to equivalent transfer function representation in the image domain
- Graphically, plant models are shown as transfer blocks

Remarks

- Not all technical processes can be modeled by linear ODEs
  $\Rightarrow$ Linearization yields linear model (see Lecture 3)
  $\Rightarrow$ Advanced synthesis methods for nonlinear systems are required (see ECE 564)