Control System Design

Lecture 3

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Elective Course in Mechatronics Engineering
Credits (2/2/3)

Webpage: http://MECE441.cankaya.edu.tr

Nonlinear System Modeling: Remarks

LTI System Operators

- Proportional gain
- Differentiation
- Integration
- Lead/lag components
- Summations

⇒ All linear operators can be represented by transfer functions

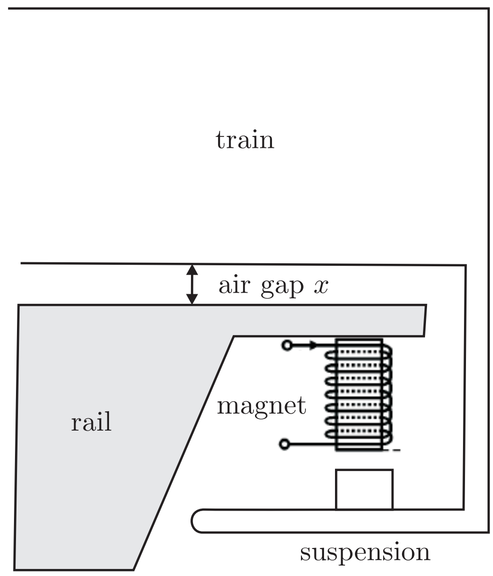
Nonlinear Systems

- Contain nonlinear system operators

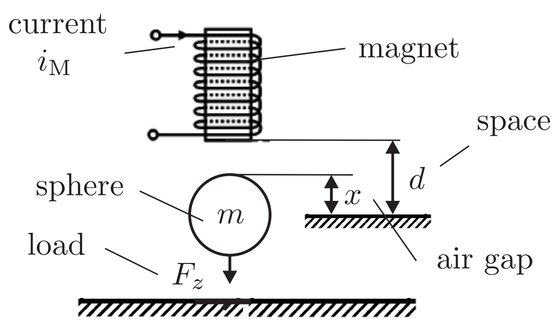
⇒ No transfer function representation

Nonlinear System Modeling: Magnetic Suspension

Schematic



Simplified Description



Simplifications

- Sphere represents vehicle
- Magnet represents suspension system

Nonlinear System Modeling: Equations

Computation

Gap 1

Nonlinear State Equations: General Form

State Equations

$$\dot{x} = f(x, u, w)$$
$$y = h(x, u)$$

Notation

- state: x, output: y, input: u, disturbance w
- f: continuous in x, u, w and additional assumptions (see for example ECE 564)
- h: continuous in x, u

Example

Gap 2

Nonlinear State Equations: Block Diagram

Example

Gap 3

Nonlinear System Modeling: Remarks

Synthesis and Analysis Techniques for Nonlinear Systems

- Beyond the scope of this lecture
  $\rightarrow$ Master-level course ECE 564
- Extensive literature
  $\rightarrow$ Alberto Isidori: "Nonlinear Control Systems", Springer, 1995 (ISBN: 3-54-019916-0)
  $\rightarrow$ Hassan K. Khalil: "Nonlinear Systems", Prentice Hall, 2002 (ISBN: 0-13-067389-7)

Set-point Linearization

- Consider system behavior in the vicinity of a given set-point
  $\rightarrow$ Assume almost linear behavior close to the set-point
  $\rightarrow$ Find a linear system model to approximate the nonlinear system

Set-Point: Definition

Set-point Definition

A set point is a stationary (non-changing) state of a system where the system output maintains a constant set-point value y_{SP}

Computation of a Set-point

- Given: y_{SP}, w_{SP}
- We want to compute x_{SP} (constant set-point value of the state) and u_{SP} (constant set-point value of the input)
- Computation

$$y_{SP} = h(x_{SP}, u_{SP})$$
$$0 = \dot{x} = f(x_{SP}, u_{SP}, w_{SP})$$

$\Rightarrow$ Solve for x_{SP}, u_{SP}

Set-Point: Example

Magnetic Suspension

Gap 4

Set-point Linearization: Description

Explanation

- Compute a „small signal“ approximation of the nonlinear system that is valid close to the set-point
- Introduce „Difference variables“ (deviation from the set-point)
- $\Delta x = x - x_{SP}$, $\Delta y = y - y_{SP}$, $\Delta u = u - u_{SP}$, $\Delta w = w - w_{SP}$

Taylor Series Expansion

$$\Delta \dot{x} = \dot{x} \approx \underbrace{f(x_{SP}, u_{SP}, w_{SP})}_{=0} + \underbrace{\frac{\partial f}{\partial x}|_{SP}}_{A} \Delta x + \underbrace{\frac{\partial f}{\partial u}|_{SP}}_{b} \Delta u + \underbrace{\frac{\partial f}{\partial w}|_{SP}}_{o} \Delta w$$

$$= A\,\Delta x + b\,\Delta u + o\,\Delta w$$

$$\Delta y \approx \underbrace{h(x_{SP}, u_{SP}) - y_{SP}}_{=0} + \underbrace{\frac{\partial h}{\partial x}|_{SP}}_{c^T} \Delta x + \underbrace{\frac{\partial h}{\partial u}|_{SP}}_{d} \Delta u = c^T\,\Delta x + d\,\Delta u$$

Set-point Linearization: Example

Example Equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -g + \frac{K_M}{m}\frac{u^2}{(d - x_1)^2} - \frac{1}{m}w$$

$$y = x_1$$

Computation

Gap 5

Set-point Linearization: Example

Computation

Gap 6

Set-point Linearization: Magnetic Suspension Example

Linearized State Equations

Gap 7

Linearization: Summary

Task

- Characterize nonlinear system behavior close to a set-point

Method

- Write system representation in terms of "difference variables"
- Use first-order Taylor series approximation for nonlinearities

Result

- We get a linear system model for the nonlinear system
- Linear methods can be used for the nonlinear system close to the set-point
- Important restriction
  $\rightarrow$ Linear model is only valid in the vicinity of the set-point