Control System Design

Lecture 4

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Mechatronics Engineering
Credits (2/2/3)

Webpage: http://mece441.cankaya.edu.tr

Reminder: Controller Types

P-Controller

$$C(s) = K_P$$

PI-Controller

$$C(s) = K_P \cdot (1 + \frac{1}{T_I\, s}) = K_p \cdot \frac{1 + s\, T_I}{s\, T_I} \quad \text{(one integrator, one zero)}$$

PID-Controller

$$C(s) = K_P \cdot (1 + \frac{1}{s\, T_I} + s\, T_D) = K_p \cdot \frac{1 + s\, T_I + s^2\, T_I\, T_D}{s\, T_I}$$

(one integrator, two zeros)

PD-Controller

$$C(s) = K_P \cdot (1 + s\, T_D) \qquad \text{(one zero)}$$

Magnitude Optimum: Prerequisites and Idea

Block Diagram

Gap 1

Prerequisites

- Stable plant: $G(s) = \dfrac{1}{A(s)} = \dfrac{1}{a_0 + \cdots + a_n s^n}$
- Controller with integrator: $C(s) = \dfrac{P(s)}{2s} = \dfrac{p_0 + \cdots + p_m s^m}{2s}$

Closed-loop Specification

- Approximate ideal reference tracking: $T(j\omega) \cong 1$

Magnitude Optimum: Idea

Computation: We want $|T(j\omega)|^2 \cong 1$

$$\begin{aligned}
|T(j\omega)|^2 &= \frac{G(j\omega)C(j\omega)}{1 + C(j\omega)G(j\omega)} \cdot \frac{G(-j\omega)C(-j\omega)}{1 + C(-j\omega)G(-j\omega)} \\
&= \frac{1}{1 + \dfrac{1}{G(j\omega)C(j\omega)}} \cdot \frac{1}{1 + \dfrac{1}{G(-j\omega)C(-j\omega)}} \\
&= \frac{1}{1 + \dfrac{2j\omega A(j\omega)}{P(j\omega)}} \cdot \frac{1}{1 + \dfrac{-2j\omega A(-j\omega)}{P(-j\omega)}}
\end{aligned}$$

Gap 2

Magnitude Optimum: Computation

Gap 3

Result

$$|T(j\omega)|^2 =$$

$$= \frac{1}{1 + 4\underbrace{\left(\frac{j\omega}{2}A(j\omega)P(-j\omega) - \frac{j\omega}{2}A(-j\omega)P(j\omega) + \omega^2 A(j\omega)A(-j\omega)\right)}_{\cong 0}/|P(j\omega)|^2}$$

Magnitude Optimum: Computation

Gap 4

$\Rightarrow$ Set first addends to 0

$\Rightarrow$ Determine appropriate controller parameters (see next slide)

- p_0, p_1 for PI-control
- p_0, p_1, p_2 for PID-control

Magnitude Optimum: Parameters

PI-Control: $P(s) = p_0 + p_1 s$

$$p_0 = a_0 \frac{a_1^2 - a_0 a_2}{a_1 a_2 - a_0 a_3}; \quad p_1 = a_1 \frac{a_1^2 - a_0 a_2}{a_1 a_2 - a_0 a_3} - a_0$$

PID-Control $P(s) = p_0 + p_1 s + p_2 s^2$

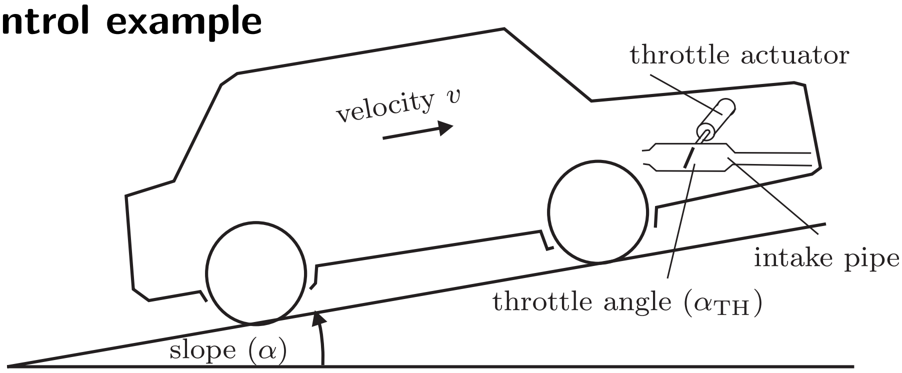
$$p_0 = \frac{1}{D} \begin{vmatrix} a_0^2 & -a_0 & 0 \\ -a_1^2 + 2a_0 a_2 & -a_2 & a_1 \\ a_2^2 + 2a_0 a_4 - 2a_1 a_3 & -a_4 & a_3 \end{vmatrix}$$

$$p_1 = \frac{1}{D} \begin{vmatrix} a_1 & a_0^2 & 0 \\ a_3 & -a_1^2 + 2a_0 a_2 & a_1 \\ a_5 & a_2^2 + 2a_0 a_4 - 2a_1 a_3 & a_3 \end{vmatrix} \qquad D = \begin{vmatrix} a_1 & -a_0 & 0 \\ a_3 & -a_2 & a_1 \\ a_5 & -a_4 & a_3 \end{vmatrix}$$

$$p_2 = \frac{1}{D} \begin{vmatrix} a_1 & -a_0 & a_0^2 \\ a_3 & -a_2 & -a_1^2 + 2a_0 a_2 \\ a_5 & -a_4 & a_2^2 + 2a_0 a_4 - 2a_1 a_3 \end{vmatrix}$$

Magnitude Optimum: Example

Vehicle control example



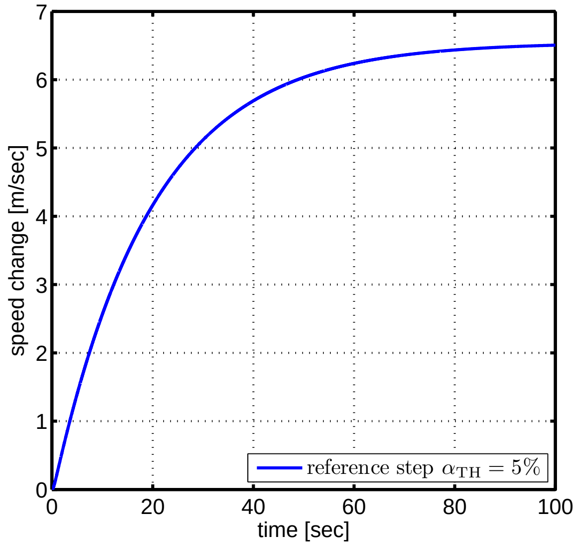
Transfer Function

- $G(s) = \dfrac{K_{\mathrm{IP}} K_{\mathrm{VD}}}{(sT_{\mathrm{IP}} + 1)(sT_{\mathrm{VD}} + 1)}$
- Input: throttle angle α_{TH}
- Output: velocity v
- Disturbance: slope α

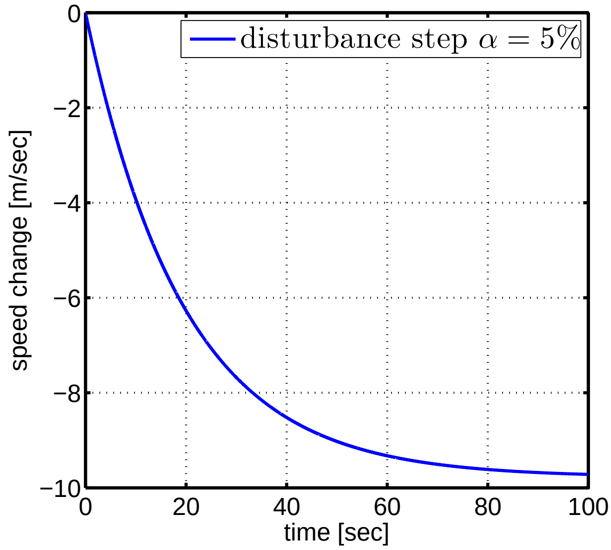
Gap 5

Magnitude Optimum: Example

Plant Input Step



Plant Disturbance Step



Parameters

$K_{\mathrm{IP}} = 7.7,\ T_{\mathrm{IP}} = 0.3,\ K_{\mathrm{VD}} = 0.17,\ T_{\mathrm{VD}} = 14.5,\ K_{\alpha} = 11.5$

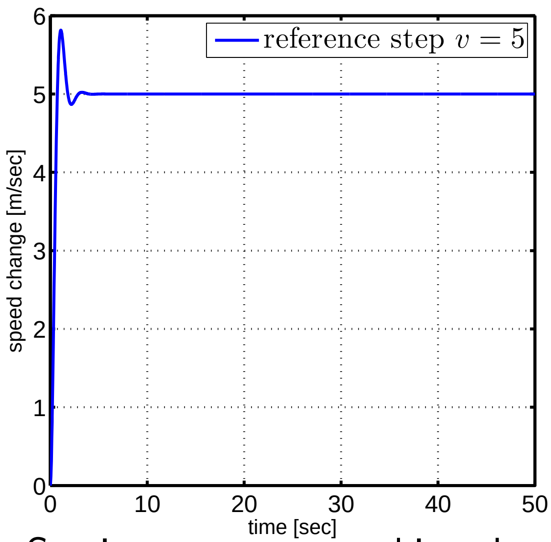
Magnitude Optimum: Example

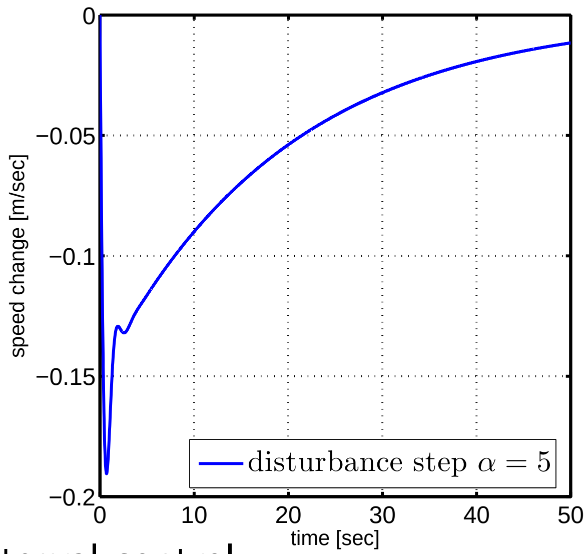
Vehicle Control

Gap 6

Magnitude Optimum: Example

Reference Step Response **Disturbance Step Response**





$\Rightarrow$ Stationary exact tracking due to integral control

$\Rightarrow$ Fast reference tracking (with overshoot)

$\Rightarrow$ Very slow disturbance rejection

Magnitude Optimum: Concluding Remarks

Usage

- Stable time-lag plants
- PI and PID controller design
- Optimize reference tracking in the feedback loop
- Plant simplifications can lead to an easier parameter computation

Limitations

- There is no stability guarantee (see Problem 10)
  $\rightarrow$ Stability problems for lightly damped plants
- Not suitable for disturbance rejection

Symmetric Optimum: Preliminaries

Prerequisites

- Stable time lag plant:

$$G(s) = \frac{K}{(1+sT_1)\cdots(1+sT_n)(1+s\tau)}$$

$\rightarrow T_1, \ldots, T_n$ are "large" time constants with $T_j >> \tau$
$\rightarrow \tau$ is a small time constant
$\rightarrow n+1$ is the system order

- Controller with integrator: $C(s) = K_P \frac{(1+sT_P)^n}{s(1+s\tau_P)^{n-1}}$, $\tau_P << T_P$

$\rightarrow \tau_P$ is small time constant for non-negative relative degree of C

Goal

- Achieve reference tracking and disturbance rejection in closed loop

Symmetric Optimum: Parameters and Example

General Parameter Choice

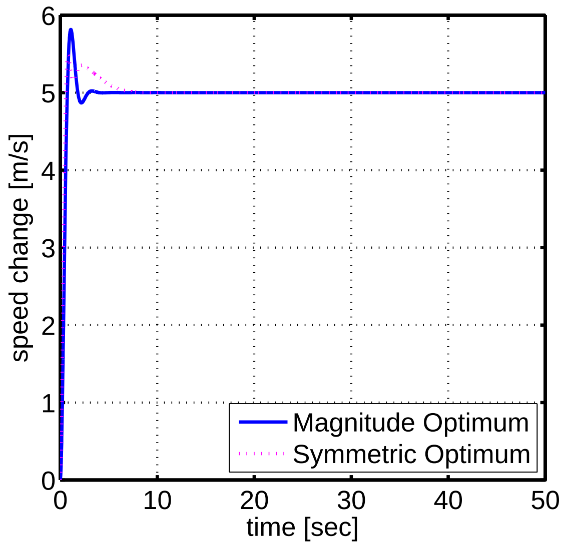
K_P	T_P	τ_P
$\frac{1}{2K\tau}\frac{T_1\cdots T_n}{(4n\tau)^n}$	$4n\tau$	$< T_P/10$

Example: Vehicle Control

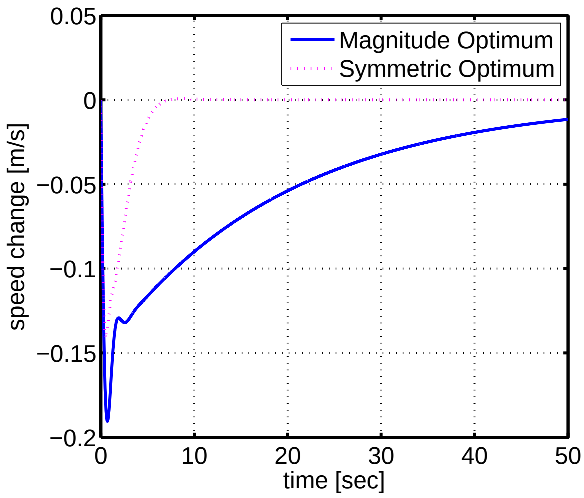
Gap 7

Symmetric Optimum: Example

Reference Step Response



Disturbance Step Response



⇒ Stationary exact tracking because of integral control

⇒ Compromise between disturbance rejection and reference tracking

Magnitude Optimum Symmetric Optimum

Symmetric Optimum: Concluding Remarks

Usage

- Time-lag plants with “large” time constants and one “small” time constant
- Design of controllers with integrator
- Addresses both reference tracking and disturbance rejection
- Design for plants with multiple small time constants $\tau_1, \ldots, \tau_m$
  $\rightarrow$ Computation with $\tau = \sum_{i=1}^{m} \tau_i$

Limitations

- Applicable to plants with certain structure
- Slower reference tracking than magnitude optimum

Klaus Schmidt Department
Department of Mechatronics Engineering – Çankaya University