Control System Design

Lecture 5

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Elective Course in Mechatronics Engineering
Credits (2/2/3)

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Youla Parametrization: Basics

Prerequisites

- Stable plant: $G(s)$, positive relative degree (proper)
- Controller transfer function: $C(s)$

Goal

- Design of a controller Q in the open loop: $T(s) = Q(s)G(s)$
- Realization of the control in the closed loop

Gap 1

Youla Parametrization: Design Method

Computation

Gap 2

$\Rightarrow C = \dfrac{Q}{1 - GQ} = \dfrac{Q}{1 - T}$ is feedback controller

Requirements

- C must have non-negative relative degree
- Closed-loop must be exponentially stable

Youla Parametrization: Results

Theorem

Consider a stable G with positive relative degree and the proposed design of $C = \dfrac{Q}{1 - QG}$. C has non-negative relative degree and the closed loop is internally stable if and only if Q has non-negative relative degree and is exponentially stable

Remarks

- Q can be interpreted as a parameter in the set of all stable transfer functions with non-negative relative degree
- Q parametrizes **all** realizable feedback controllers C that internally stabilize the control loop and **all** possible open loops $T = QG$

Youla Parametrization: Plant Inversion

- Ideal reference tracking for $T(j\omega) = Q(j\omega)G(j\omega) \equiv 1$
  $\Rightarrow$ Requires $Q(s) = G(s)^{-1}$
- Issue 1: Leads to negative relative degree of $Q(s)$
  $\Rightarrow$ Specify appropriate relative degree of $T(s)$, for example
  $T(s) = \dfrac{1}{(1 + s\tau)^r}$ (choice of r in Problem 14 **b.**)
- Issue 2: No exponential stability of $Q(s)$ if $G(s)$ has instable zeros
  $\Rightarrow$ Write $G(s) = \dfrac{B^+(s) \cdot B^-(s)}{A(s)}$ (instable zeros in $B^+(s)$)
  $\Rightarrow$ Only zeros in B^- can be compensated by Q
  $\Rightarrow$ Open loop design: $T(s) = \dfrac{B^+(s)}{(1 + s\tau)^r} \Rightarrow Q(s) = \dfrac{A(s)}{B^-(s)(1 + s\tau)^r}$
- Remark: Keeping instable zeros in $T(s)$ is general requirement for the basic feedback loop

Youla Parametrization: Vehicle Control Example

Vehicle control

Gap 3

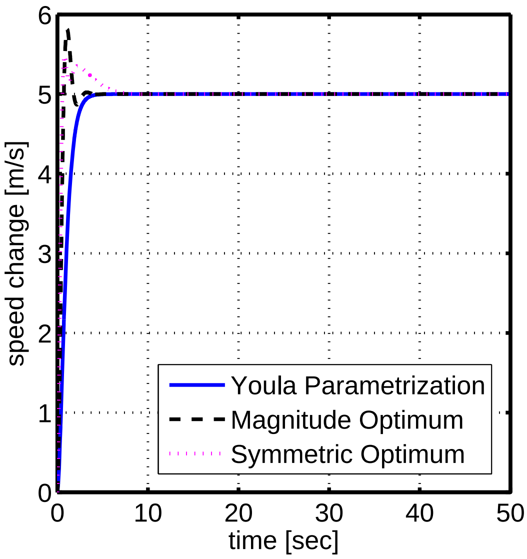
Youla Parametrization: Vehicle Control Example

Vehicle control

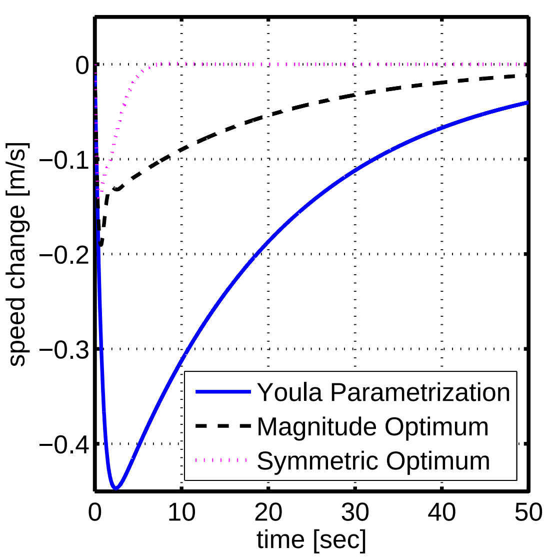
Gap 4

Youla Parametrization: Vehicle Control Example

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Fast dynamics due to fast specified closed-loop poles

$\Rightarrow$ Zero steady-state error as specified in $T(s)$

$\Rightarrow$ Slow disturbance response due to zeros in $S(s)$

Youla Parametrization: Temperature Example

Temperature control

Gap 5

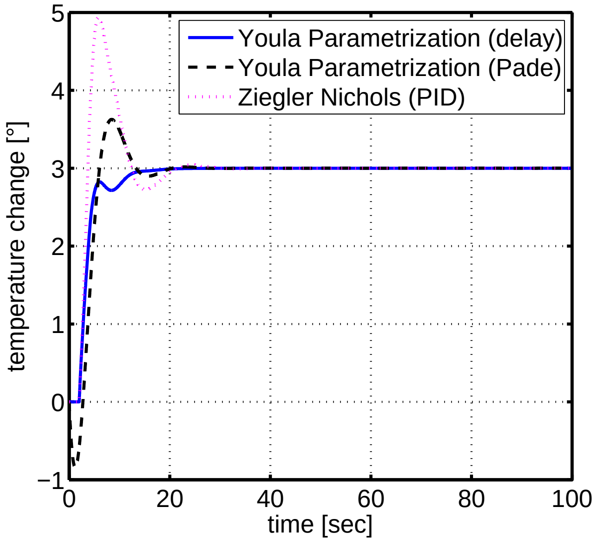
Youla Parametrization: Temperature Example

Temperature control

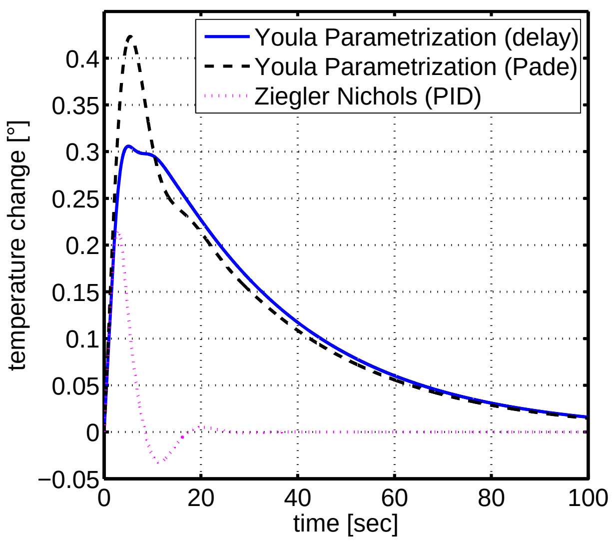
Gap 6

Youla Parametrization: Output Trajectories

Reference Step Response



Disturbance Step Response



⇒ Good reference tracking due to specification of $T(s)$

Youla Parametrization: Concluding Remarks

Usage

- Stable plants with positive relative degree
- Linear controller design
- Directly specify $T(s) = G(s)Q(s)$
- $Q(s)$ parametrizes all suitable feedback controllers $C(s) = \dfrac{Q(s)}{1 - T(s)}$

Limitations

- Specification of $T(s)$ can lead to bad disturbance rejection
- Appropriate relative degree of $T(s)$ has to be chosen
- Applies only to stable plants