

# Control System Design

## Lecture 6

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Mechatronics Engineering  
Credits (2/2/3)

Webpage: <http://mece441.cankaya.edu.tr>

## Internal Model Control (IMC): Youla Parametrization

### Prerequisites for Youla Parametrization

- Stable plant:  $G(s)$ , positive relative degree

### Goal

- Design  $T(s) = Q(s)G(s)$
- Determine  $Q(s)$  stable and with non-negative relative degree
- Compute controller  $C(s) = \frac{Q(s)}{1-T(s)}$

### Block Diagram Representation of Youla Parametrization

Gap 1

# Internal Model Control (IMC): Equivalent Representation

## IMC-Control Loop

Gap 2

⇒ No feedback if the plant model is exact and there is no disturbance

### Behavior for exact Plant Model

- $T = QG$
- $S = 1 - QG$

# Internal Model Control (IMC): Non-exact Plant Model

## Behavior for non-exact Plant Model

$$T = \frac{C\hat{G}}{1 + C\hat{G}} = \frac{\frac{Q}{1-QG}\hat{G}}{1 + \frac{Q}{1-QG}\hat{G}} = \frac{Q\hat{G}}{1 + Q(\hat{G} - G)}$$

$$S = \frac{1}{1 + \frac{Q}{1-QG}\hat{G}} = \frac{1 - QG}{1 + Q(\hat{G} - G)}$$

### Robustness of IMC

- Assume  $\hat{G} = G(1 + \delta G)$
- Assume upper bound estimate for uncertainty  $|\delta G(s)| < B(s)$
- Stability of the IMC-loop requires  $|Q(j\omega)G(j\omega)B(j\omega)| < 1$  for all  $\omega$

# Smith Predictor: Time-Delay Plant

## Transfer function

$$G(s) = \tilde{G}(s)e^{-s\tau}$$

- Time-delay:  $\tau$

## Example

Gap 3

# Smith Predictor: Basics

## Prerequisites

- Delay plant:  $G(s) = \tilde{G}(s)e^{-s\tau}$ ,  $\tilde{G}(s)$  rational in  $s$

## Goal

- Design of preliminary controller  $\hat{C}(s)$  without delay component  $e^{-s\tau}$
- Consideration of delay in particular controller structure

Gap 4

# Smith Predictor: Properties

## Controller Structure

- Compensate delayed plant reaction by parallel model  $\hat{G}(s)e^{-s\tau}$  (ideally  $\tilde{G}(s) = \hat{G}(s)$  and  $y = \hat{y}$ )
- Ideally, control loop is formed by  $\hat{C}(s)$  and  $\hat{G}(s)$  (predicts reaction of plant without delay)  
⇒ Controller design for  $\hat{G}(s)$
- Plant output  $y$  follows internal output  $\tilde{y}$  with delay  $\tau$

## Equivalent Loop

Gap 5

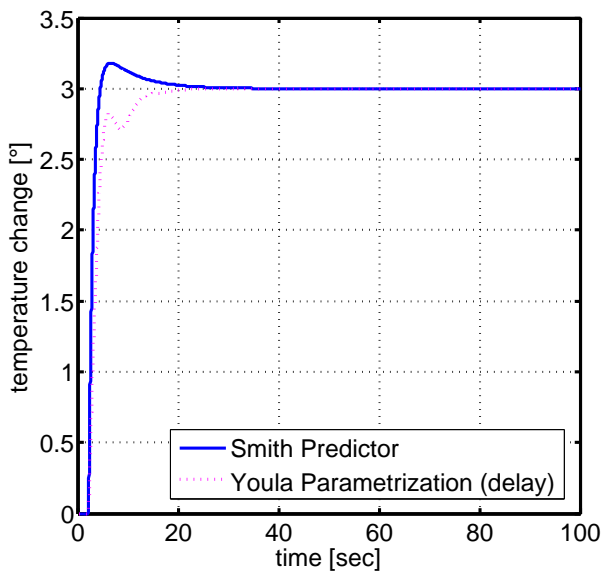
# Smith Predictor: Example

## Temperature control

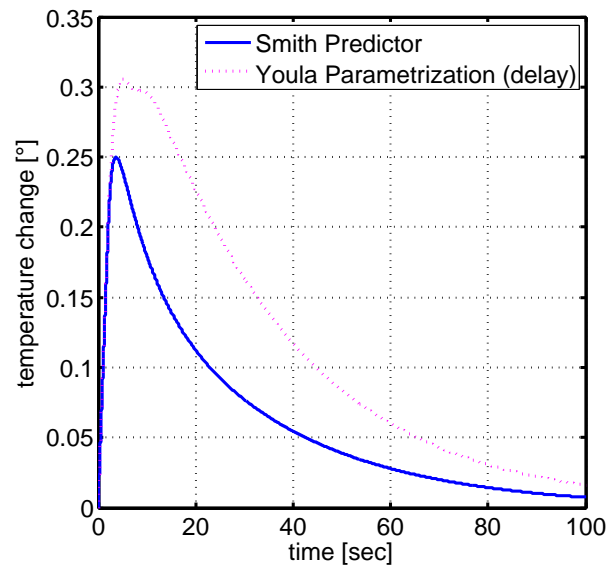
Gap 6

# Youla Parametrization: Output Trajectories

## Plant Input Step



## Plant Disturbance Step



⇒ Reference tracking follows designed closed loop with delay 2

⇒ Disturbance rejection depends on design of  $\hat{C}$

# Smith Predictor: Concluding Remarks

## Usage

- Stable delay plants
- Standard design for delay-free part of the plant
- “Delay outside the loop”

## Limitations

- Not applicable to instable plants
- Possible degradation of disturbance rejection due to controller structure