Control System Design

Lecture 7

Associate Prof. Dr. Klaus Schmidt

Department of Mechatronics Engineering – Çankaya University

Elective Course in Mechatronics Engineering
Credits (2/2/3)

Webpage: http://mece441.cankaya.edu.tr

Motivation: Basic Setup

Prerequisites

- General plant: $G(s) = \dfrac{B(s)}{A(s)} = \dfrac{b_0 + \cdots + b_n s^n}{a_0 + \cdots + a_n s^n}$ ($A(s)$, $B(s)$ coprime) without time delay

Controller

- $C(s) = \dfrac{P(s)}{L(s)} = \dfrac{p_0 + \cdots + p_m s^m}{l_0 + \cdots + l_m s^m}$

Observation

- Each zero of $AL + BP$ is a pole of at least one sensitivity

$$T = \frac{GC}{1+GC} = \frac{BP}{AL+BP} \quad S_{\mathrm{i}} = \frac{G}{1+GC} = \frac{BL}{AL+BP}$$
$$S = \frac{1}{1+GC} = \frac{AL}{AL+BP} \quad S_{\mathrm{u}} = \frac{C}{1+GC} = \frac{AP}{AL+BP}$$

Motivation: Goal

Goal

- Place closed-loop poles at pre-specified pole locations
  → Closed-loop polynomial $R(s) = r_0 + r_1\, s + \cdots + r_p s^p$
  → Stability, closed-loop dynamics

Design Problem

Given a plant $G(s) = \dfrac{B(s)}{A(s)}$ *and a desired closed-loop denominator polynomial* $R(s) = r_0 + \cdots + r_p s^p$*, find a controller transfer function* $C(s) = \dfrac{P(s)}{L(s)}$ *such that* $A(s)L(s) + B(s)P(s) = R(s)$.

Pole Placement: Properties

Basic Properties

Gap 1

Pole Placement: Basic Procedure

Design Idea

- Use controller transfer function of order $m \geq n-1$:

$$C(s) = \frac{P(s)}{L(s)} = \frac{p_0 + p_1\, s + \cdots + p_m\, s^m}{l_0 + l_1\, s + \cdots + l_m\, s^m}$$

- Write down the design equation

$$R(s) = A(s)L(s) + B(s)P(s)$$

- Compute the free parameters $l_0, \ldots, l_m$ and $p_0, \ldots, p_m$ by comparison of coefficients

Result

- P-control for plants with $n = 1$
- Lead/lag controllers for plants with $n > 1$

Pole Placement: Simple Example

Computation

Gap 2

Pole Placement: Simple Example

Computation

Gap 3

Pole Placement: General Procedure

Comparison of Coefficients

$$A(s)L(s) + B(s)P(s) = R(s)$$

$$(a_0 + \cdots a_n s^n)(l_0 + \cdots l_m s^m) + (b_0 + \cdots b_n s^n)(p_0 + \cdots p_m s^m) = R(s)$$

Gap 4

Pole Placement: Solution

Evaluation

Gap 5

Pole Placement: Solution

Evaluation

Gap 6

Pole Placement: Solution

General Solution Equation for $p = 2n-1$

$$\underbrace{\left[\begin{array}{ccc|ccc} a_n & 0 \cdots & 0 & b_n & 0 \cdots & 0 \\ \vdots & \ddots & 0 & \vdots & \ddots & 0 \\ a_1 & \cdots & a_n & b_1 & \cdots & b_n \\ \hline a_0 & \cdots & a_{n-1} & b_0 & \cdots & b_{n-1} \\ 0 & \ddots & \vdots & 0 & \ddots & \vdots \\ 0 \cdots 0 & & a_0 & 0 \cdots 0 & & b_0 \end{array}\right]}_{\text{Sylvester Matrix } M} \underbrace{\left[\begin{array}{c} l_{n-1} \\ \vdots \\ l_0 \\ \hline p_{n-1} \\ \vdots \\ p_0 \end{array}\right]}_{v} = \underbrace{\left[\begin{array}{c} r_{2n-1} \\ \vdots \\ r_n \\ \hline r_{n-1} \\ \vdots \\ r_0 \end{array}\right]}_{r}$$

Matrix Inversion

- M is invertible if $A(s)$ and $B(s)$ do not have common zeros $\surd$
  $\Rightarrow v = M^{-1} r$
- Note: choice of closed loop poles according to performance criteria

Pole Placement: Example

Vehicle Control Example

Gap 7

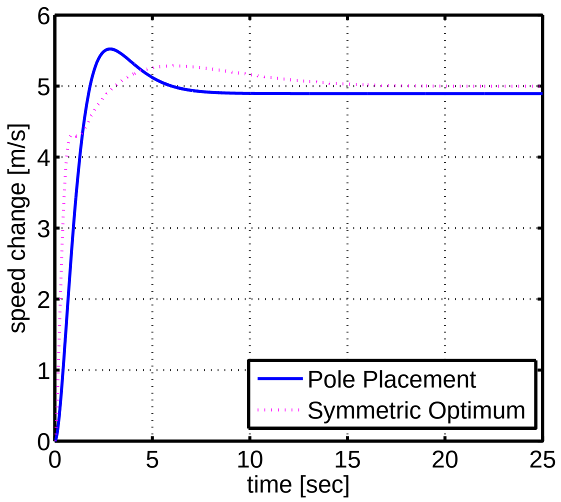
Pole Placement: Example

Vehicle Control Example

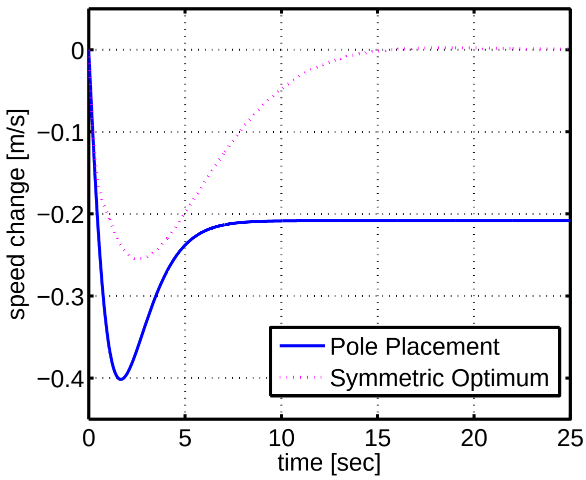
Gap 8

Pole Placement: Simulation

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Fast dynamics due to fast closed-loop poles

$\Rightarrow$ No stationary exact tracking due to lack of integral control

Pre-filter: Problem

Observed Problem

- Large overshoot in reference step response due to zeros in $T(s)$

Gap 9

Decompose Numerator of $T(s)$

- $T(s) = \dfrac{V^+(s)V^-(s)}{R(s)}$
- $V^+(s)$: polynomial with instable zeros of $T(s)$ (RHP)
- $V^-(s)$: polynomial with stable zeros of $T(s)$ (OLHP)

$\Rightarrow$ We want to compensate the stable zeros in $T(s)$

Pre-filter: Design

Filter Transfer Function

$$F(s) = \frac{R(0)}{V^-(s)\,V^+(0)}$$

$\Rightarrow$ Compensation of stable zeros in $T(s)$

Feedback Loop with Pre-filter

Gap 10

Pre-filter: Example

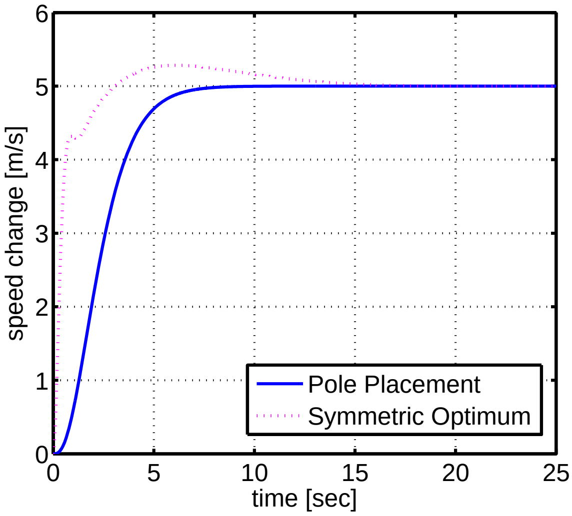
Vehicle Control Example with Pre-filter

Gap 11

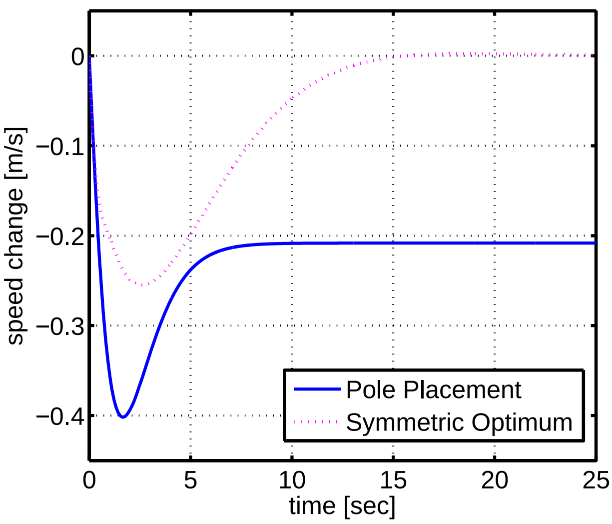
Motivation Pole Placement Pre-filter

Pre-filter: Simulation

Reference Step Response



Disturbance Step Response



⇒ Fast reference step response without overshoot

⇒ No change in the disturbance rejection (filter outside the loop)

Klaus Schmidt Department
Department of Mechatronics Engineering – Çankaya University