Control System Design

Lecture 8

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Elective Course in Mechatronics Engineering
Credits (2/2/3)

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Reminder: Pole Placement

Prerequisites

- General plant: $G(s) = \dfrac{B(s)}{A(s)} = \dfrac{b_0 + \cdots + b_n s^n}{a_0 + \cdots + a_n s^n}$ ($A(s)$, $B(s)$ coprime) without time delay

Controller

- $C(s) = \dfrac{P(s)}{L(s)} = \dfrac{p_0 + \cdots + p_m s^m}{l_0 + \cdots + l_m s^m}$

Observation

- Each zero of $AL + BP$ is a pole of at least one sensitivity

$$T = \frac{GC}{1+GC} = \frac{BP}{AL+BP} \quad S_\mathrm{i} = \frac{G}{1+GC} = \frac{BL}{AL+BP}$$
$$S = \frac{1}{1+GC} = \frac{AL}{AL+BP} \quad S_\mathrm{u} = \frac{C}{1+GC} = \frac{AP}{AL+BP}$$

Reminder: Solution

Task

- Place closed-loop poles at pre-specified pole locations
  $\rightarrow$ Closed-loop polynomial $R(s) = r_0 + r_1\, s + \cdots + s^p$

Procedure

- Choose controller degree as $m \geq n-1$ (usually $m = n-1$)
- Compute $p = m + n = n - 1 + n = 2n - 1$
- Find free parameters $l_0, \ldots, l_m$ and $p_0, \ldots, p_m$ from design equation
  $A(s)\, L(s) + B(s)\, P(s) = R(s)$
- Use pre-filter to compensate stable zeros in $T(s)$

Disadvantage

- Controller without integrator
  $\rightarrow$ steady-state error is non-zero

Feedback Loop Analysis: Steady-state Error

Reference and Disturbance Steps

- Steady-state error is zero if controller has integrator
  $\rightarrow C(s) = \dfrac{p_0 + \cdots + p_m s^m}{s\,(l_0 + l_1\, s + \cdots + l_{m-1} s^{m-1})}$

Sinusoidal Reference and Disturbance Signals

- Recall the frequency response for stable transfer functions $G(s)$ and sinusoidal input $u(t) = u_0 \sin(\omega_0\, t)$

$$y(t) = u_0\, |G(j\omega_0)| \sin(\omega_0\, t + \angle(G(j\omega_0)))$$

- Reference tracking for $r(t) = r_0\, \sin(\omega_0\, t)$
  $\rightarrow T(j\omega_0) = 1$
- Disturbance rejection for $d(t) = d_0\, \sin(\omega_0\, t)$
  $\rightarrow S(j\omega_0) = 0$ and $S_i(j\omega_0) = 0$

Feedback Loop Analysis: Properties

Computation

Gap 1

$\Rightarrow$ We should include the factor s or $s^2 + \omega_0^2$ in the controller denominator

Extended Pole Placement: Goal

Goal

- Place closed-loop poles at pre-specified pole locations
  $\rightarrow$ Stability, closed-loop dynamics
- Place some controller poles at pre-specified locations
  $\rightarrow$ zero steady-state error for open-loop pole at $s = 0$

Design Problem

Given a plant $G(s) = \frac{B(s)}{A(s)}$, a desired closed-loop denominator polynomial $R(s) = r_0 + \cdots + r_p s^p$ and a polynomial $\hat{L}(s)$ of degree κ, find a controller $C(s) = \frac{P(s)}{\hat{L}(s)\,L(s)}$ such that $A(s)\hat{L}(s)\,L(s) + B(s)P(s) = R(s)$.

Extended Pole Placement: Properties

Basic Properties

Gap 2

Extended Pole Placement: Procedure

Design Idea

- Consider polynomial $\hat{L}(s)$ as part of the plant denominator

$$\hat{A}(s) = A(s)\hat{L}(s) \text{ (the coefficients of } \hat{L}(s) \text{ are fixed)}$$

- Design equation

$$R(s) = \hat{A}(s)L(s) + B(s)P(s)$$

$\Rightarrow$ Compute the free parameters $l_0, \ldots, l_{m-\kappa}$ and $p_0, \ldots, p_m$
$\Rightarrow$ Minimum controller order: $m \geq n + \kappa - 1$

Coefficient Matching

$$\hat{A}(s)L(s) + B(s)P(s) = R(s)$$

$$(\hat{a}_0 + \cdots \hat{a}_{n+\kappa}s^{n+\kappa})(l_0 + \cdots l_{m-\kappa}s^{m-\kappa}) + (b_0 + \cdots b_n s^n)(p_0 + \cdots p_m s^m) = R(s)$$

Extended Pole Placement: Solution

Evaluation

Gap 3

Extended Pole Placement: Solution

Solution Method

- Determine controller coefficients by comparison of coefficients

Control with Integrator

- Specify one controller pole at $s = 0$: $\hat{L}(s) = s$ and $\kappa = 1$
- Controller degree: $m = n + \kappa - 1 = n$
  $\Rightarrow$ PI-control for plant with degree 1: $m = 1 + 1 - 1 = 1$
  $\Rightarrow$ PID-control for plant with degree 2: $m = 2 + 1 - 1 = 2$
- Higher order plants lead to controllers that cannot be realized by classical PID control

Controller for Sinusoidal References/Disturbances (frequency ω_0)

- Specify controller poles at $s = \pm j\omega_0$: $\hat{L}(s) = s^2 + \omega_0^2$ and $\kappa = 2$
- Controller degree: $m = n + \kappa - 1 = n + 1$

Extended Pole Placement: Example

Vehicle Control Example with Integral Control

Gap 4

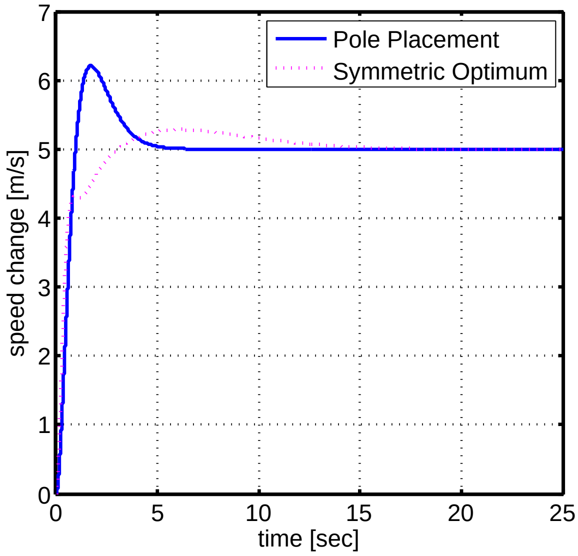
Extended Pole Placement: Example

Vehicle Control Example with Integral Control

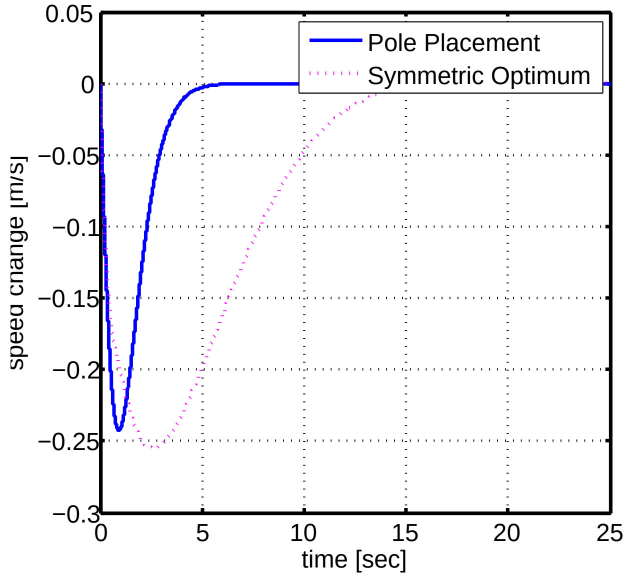
Gap 5

Extended Pole Placement: Simulation

Reference Step Response



Disturbance Step Response

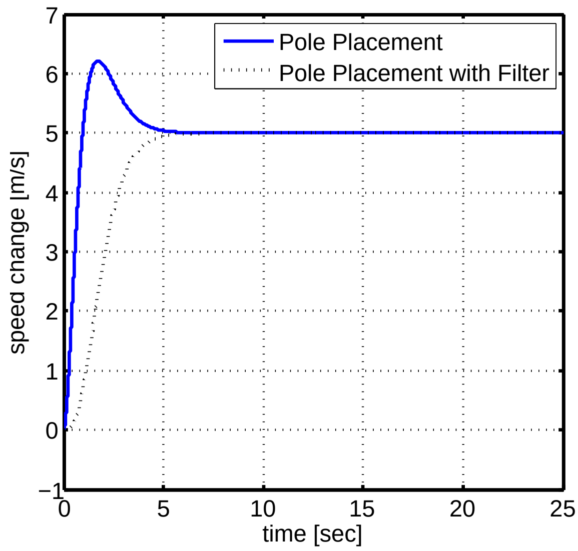


⇒ Fast reference step response and disturbance rejection

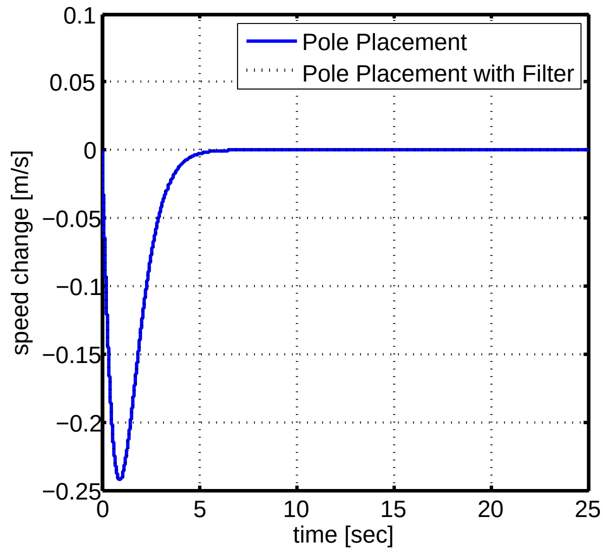
⇒ Steady-state error is zero but overshoot

Extended Pole Placement: Simulation with Pre-filter

Reference Step Response



Disturbance Step Response



⇒ Fast reference step response without overshoot

⇒ No change in the disturbance rejection (filter outside the loop)

Pole Placement: Solution for $p = 2n + \kappa - 1$

Solution Equation for $p = 2n + \kappa - 1$

$$
\underbrace{\left[\begin{array}{ccc|ccccc}
\hat{a}_{n+\kappa} & 0 \cdots & 0 & b_n & 0 \cdots & 0 & 0 & 0 \\
\vdots & \ddots & 0 & \vdots & \ddots & 0 & 0 & 0 \\
\hat{a}_{\kappa+1} & \cdots & \hat{a}_{n+\kappa} & b_1 & \cdots & b_n & 0 & 0 \\
\hline
\hat{a}_{\kappa} & \cdots & \hat{a}_{n+k-1} & b_1 & \cdots & \cdots & b_n & \cdots 0 \\
\vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \cdots \ddots & 0 \\
\hat{a}_1 & \cdots & \hat{a}_n & 0 \cdots & b_0 & b_1 & \cdots & b_n \\
\hat{a}_0 & \cdots & \hat{a}_{n-1} & 0 & 0 & b_0 & \cdots & b_{n-1} \\
0 & \ddots & \vdots & 0 & 0 & 0 & \ddots & \vdots \\
0 \cdots 0 & & \hat{a}_0 & 0 & 0 & 0 \cdots & 0 & b_0
\end{array}\right]}_{\text{Sylvester Matrix } M}
\underbrace{\left[\begin{array}{c}
l_{n-1} \\ \vdots \\ l_0 \\ \hline p_{n+\kappa-1} \\ \vdots \\ p_0
\end{array}\right]}_{v}
=
\underbrace{\left[\begin{array}{c}
r_{2n+\kappa-1} \\ \vdots \\ r_{n+\kappa} \\ \hline r_{n+\kappa-1} \\ \vdots \\ r_0
\end{array}\right]}_{r}
$$